

This tutorial helps for assignment 3, helping you become more comfortable with transfer functions, and assists your understanding of feedback controllers and block diagrams.

**Question 1 [8]**

The following questions refer to a first order system:  $G_p(s) = \frac{y(s)}{u(s)} = \frac{\text{gain}}{(\text{time constant})s + 1} = \frac{K_p}{\tau s + 1}$

1. What are the units for a time constant?
2. What are the units for a gain?
3. Can a process have a negative gain?
4. Give an example of a process that has a time constant of zero.
5. Can a process have a zero gain? Explain why or why not.

**Question 2 [8]**

1. Draw the following input,  $f(t)$ , as a function of time (this is the actual input to a batch reaction, commonly used in the pharmaceutical industry, or many companies with batch systems).
  - Start at 25 units at time zero; ramp up by 1 unit per minute; keep ramping up for 60 minutes.
  - Remain at the current position for 30 minutes, keeping the input constant.
  - Jump suddenly down to the starting point.
2. Derive the Laplace transform of  $F(s) = \mathcal{L}[f(t)]$ .

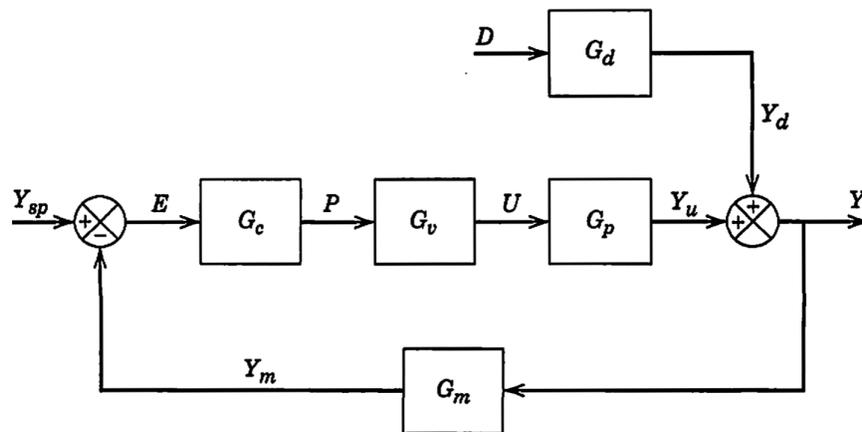
### Question 3

Below is the standard feedback control diagram for any process,  $G_p$ , which has an input  $U$  and output  $Y_u$ . There are other transfer functions, such as  $G_c$ , which takes the set point value,  $Y_{sp}$  and *subtracts* from it the measurement from the sensor,  $Y_m$ . The controller then issues a signal to the valve to open or close, via the value  $P$ . The valve also has a (fast) transfer function,  $G_v$ , which creates the actual process input signal  $U$ .

Our measurement on the process,  $Y$ , is made up of two parts,  $Y_d$  and  $Y_u$ . The transfer function  $G_m$  represents the dynamics of the sensor (measurement), a similar idea to the transfer function for the thermocouple in question 2 of assignment 3.

Each of the transfer functions for the *systems* are in Laplace form, i.e.  $G_p$  should actually be written as  $G_p(s)$ . Each of the transfer functions for the *signals* are in Laplace form, i.e.  $Y$  should actually be written as  $Y(s)$ .

Today's task is simple: become comfortable with the notation in the block diagram - we are going to see and use this many times, which is why we drop off the  $(s)$  parts on the *systems* and *signals*.



1. Derive an expression which gives the output  $Y(s)$  for a given valve input signal  $P(s)$  and a given disturbance signal  $D(s)$ . Your answer should be  $Y(s) = \dots$
2. Derive a single transfer function which gives the output  $Y(s)$  for a given input  $Y_{sp}(s)$ . For this part of the question, assume input  $D$ , and consequently  $Y_d$  are both zero (for today). Your answer:  $\frac{Y(s)}{Y_{sp}(s)} = \dots$